Shadow Series in the Munsell System

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Abstract

Using an inversion of the Munsell renotation, this paper calculates that a colour's shadow series is approximately a straight line in the Munsell system. The line starts at the colour's Munsell specification and ends about one value step below N0, on the neutral axis. The colour's hue in shadow shifts slightly towards the yellow part of the spectrum. The calculations suggest that ideal black belongs at about N(-1) in the Munsell system, rather than at N0, if equality of perceptual steps is to be maintained. Similarly, ideal white should be slightly lighter than N10.

Keywords: Munsell, shadow series, renotation, Evans, ideal black, ideal white

1 Introduction

Shadows are ubiquitous when viewing physical objects. In any realistic setting, some parts of an object receive more light than other, more shaded parts. In a black and white painting or photograph, the shadowed parts of an object, whose local colour is assumed not to vary, are a darker grey than the more strongly lit parts. In a representation in colour, there must also be a relationship, which this article derives, between the shadowed and lit parts.

Suppose that the colour when lit has Munsell coordinates HV/C. For example, HV/C could be 6GY8/14. Locate the lit colour in the Munsell section of constant hue H, as shown in Figure 1. The left axis is a vertical line of neutral greys, numbered from N1 through N9. Continue the neutral axis downward, to where the value would be negative. Draw a line from the value -1.6 on the neutral axis, to the center of the square containing 6GY8/14. The shadow colours of 6GY8/14 fall approximately on this line. The value on the neutral axis is not always -1.6, but usually varies between 0 and -2, depending on the original colour.

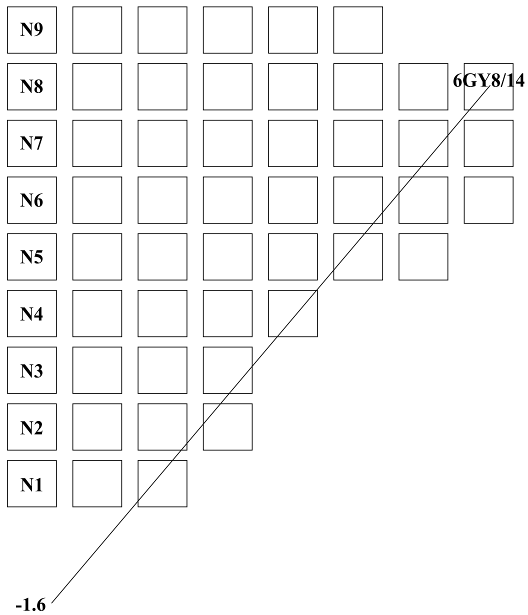

Figure 1: Munsell Hue Sheet for 6GY

This paper derives shadow series in the Munsell system. An exhaustive computer calculation inverts the Munsell renotation[1], which is an empirical conversion of Munsell coordinates to (x, y, Y) coordinates. Its inverse[2–5] converts the (x, y, Y) system to the Munsell system. When plotted in the Munsell system, the calculated shadow colours are observed to fall approximately along straight lines, which generally cross the neutral axis below the value 0, but above the value -2. Rather than relying on visual inspection, the linearity is demonstrated rigorously. Though hues are approximately constant in a shadow series, the calculations do identify a slight, systematic tendency towards the yellow part of the spectrum as a colour is seen more deeply in shadow.

An examination of the form of shadow series in the Munsell system suggests the hypothesis that ideal black and white should not have their current Munsell coordinates of N0 and N10. Without physical samples, there is no empirical way to determine how many perceptual value steps ideal black is below N1, nor how many steps ideal white is above N9. Extrapolating the linear shadow series beyond the renotation data, which extends from values 1 through 9, suggests that ideal black is probably near N(-1), while ideal white is slightly lighter than N10, in terms of perceptual value steps.

Much of the current paper can be seen as a rigorous refinement and extension of Ralph Evans's investigations in the late 1960s[6,7]. In his work, Evans made some simplifying assumptions which, the current paper shows, can be demonstrated directly, or removed entirely. To maintain his model, for example, Evans knowingly disregarded some data for Munsell values 1 and 9. In fact, this data can be included, if Evans's model is reformulated. By today's standards, a lack of computing power hampered Evans. Our increased calculational capabilities can eliminate much of the simplification that was necessary in his day.

Though some early formulations of the Munsell system incorporated shadow series implicitly, this paper argues that today's Munsell system should be viewed as a purely perceptual system. In this light, the current calculations relate a human phenomenon (perception) to a physical phenomenon (shadow series). Though it turns out that shadow series take a surprisingly simple linear form, no explanation for that form is attempted here.

2 Derivation of Shadow Series

Suppose that an object has a constant, non-glossy, local colour. *Local colour* is understood to be the object's colour, insofar as that colour is a result of the object's reflectance spectrum. Denote the reflectance spectrum by $r(\lambda)$, where λ is a wavelength varying from approximately 360 to 760 nm, and r takes on values between 0 and 100%. Suppose also that the only light illuminating this object also has an unvarying spectral composition, $s(\lambda)$. The intensity of $s(\lambda)$ is assumed to produce ordinary daytime, or photopic, vision.

Because of the object's orientation relative to the light source (or sources), and because of diffusion and interfering objects, some parts of the object receive more light than other parts, which are in shadow. For example, a part in light might receive light of spectral composition $k_1 s(\lambda)$, while a part in shadow receives light of spectral composition $k_2 s(\lambda)$, where k_1 and k_2 are both between 0 and 1, and $k_1 > k_2$. Note that only the intensity of the illumination changes, not its relative spectral power density (SPD).

Relative CIE coordinates[8] can be calculated for the object in light:

$$X_1 = \frac{\int_{360}^{760} \bar{x}(\lambda) r(\lambda) k_1 s(\lambda) d\lambda}{\int_{360}^{760} \bar{y}(\lambda) s(\lambda) d\lambda}, \tag{1}$$

$$Y_1 = \frac{\int_{360}^{760} \bar{y}(\lambda) r(\lambda) k_1 s(\lambda) d\lambda}{\int_{360}^{760} \bar{y}(\lambda) s(\lambda) d\lambda}, \tag{2}$$

$$Z_1 = \frac{\int_{360}^{760} \bar{z}(\lambda) r(\lambda) k_1 s(\lambda) d\lambda}{\int_{360}^{760} \bar{y}(\lambda) s(\lambda) d\lambda}, \tag{3}$$

where $\bar{x}, \bar{y}$, and $\bar{z}$ are the standard CIE colour-matching functions. Equations (1) through (3) can be transformed to chromaticity coordinates:

$$x_1 = \frac{X_1}{X_1 + Y_1 + Z_1}, \tag{4}$$

$$y_1 = \frac{Y_1}{X_1 + Y_1 + Z_1}, \tag{5}$$

$$Y_1 = Y_1. \tag{6}$$

The CIE coordinates for the object in shadow can be calculated by replacing k_1 with k_2 in Equations (1) through (3). We then get the relationships

$$X_2 = \frac{k_2}{k_1} X_1, \tag{7}$$

$$Y_2 = \frac{k_2}{k_1} Y_1, \tag{8}$$

$$Z_2 = \frac{k_2}{k_1} Z_1. \tag{9}$$

(x, y, Y) for the shadow colour can be calculated from the formulas in Equations (4) through (6), replacing the subscript 1 with the subscript 2. Making this replacement, and combining the results with Equations (7) through (9) gives

$$x_1 = x_2, \tag{10}$$

$$y_1 = y_2, \tag{11}$$

$$Y_1 = \frac{k_1}{k_2} Y_2. \tag{12}$$

Equations (10) through (12) show that a colour's chromaticity coordinates are the same, whether the colour is in light or shadow. The relative luminance, on the other hand, is less for a colour in shadow, than for that same colour in light. A shadow series is a sequence of colours, that results from casting an increasing amount of shadow on the first colour. The colours in a shadow series share the same x and y, but Y becomes progressively smaller.

Equations (10) to (12) make it straightforward to generate shadow series in (x, y, Y) coordinates: fix x and y, and let Y vary. The smallest value for Y, in theory at least, is 0. For each fixed x and y, there is a maximum Y value that produces a physically realizable colour. A colour whose Y value is maximal is called an optimal colour. Optimal colours vary with the illuminant. Tables of optimal colours have been calculated for standard illuminants such as A and D65[9], and C[10].

A set of shadow series was generated for analysis. Each shadow series began with an optimal colour for Illuminant C, which is the standard illuminant for the Munsell system[1]. The optimal colours used were those listed by David MacAdam[10]. The Munsell renotation data does not always extend to the MacAdam limits, so the optimal colour could sometimes not be inverted. In this case, the Y value for the optimal colour was reduced until its inverse could be calculated. The series analyzed here consisted of all Y values, less than the maximal Y, which corresponded to an integer Munsell value. For example, a luminance factor of $Y = 30\%$ occurs when the Munsell value, V, is about 6. An invertible quintic polynomial, Eq. (2) of ASTM D1535-08[4], was used to convert between V and Y. It is only necessary to consider shadow series of optimal colours, becaues the shadow series of a non-optimal colour, (x, y, Y), is a subset of the shadow series of the optimal colour (x, y, Y_{max}), where Y_{max} is the largest Y value possible for a colour of chromaticity (x, y).

3 Shadow Series in the Munsell System

An inverse[5] to the Munsell renotation was used to convert the shadow series from xyY coordinates to Munsell coordinates. Strictly speaking, the Munsell renotation and its inverse only apply when $s(\lambda)$ has the same relative SPD as illuminant C. As long as $s(\lambda)$ is generally broadband, however, chromatic adaptation will insure that the inverse renotation gives a good approximation to the Munsell coordinates. The Munsell renotation presents xyY coordinates only for colours of integer Munsell value. Extending or inverting the renotation for non-integer values requires interpolation, which different researchers might perform differently, particularly for values less than 1 or greater than 9. To mitigate these differences, the shadow series that were analyzed consist of only samples of integer Munsell values. In addition, a shadow series was analyzed only if it contained at least three colours. In all, 182 shadow series, comprising 1202 colour samples, were calculated and analyzed.

Each shadow series was plotted on two Munsell graphs, one showing value and chroma, and the other showing hue and chroma. Figures 2 and 3 plot the shadow series for 6GY8/14. In Figure 2, the shadow series appears as a slightly irregular line, starting at value 8 and chroma 14. The values in the shadow series progressively decrease as the colours get darker. It can be seen that the chroma decreases, too. Consecutive points have been joined by a line. A least squares line was fit to the

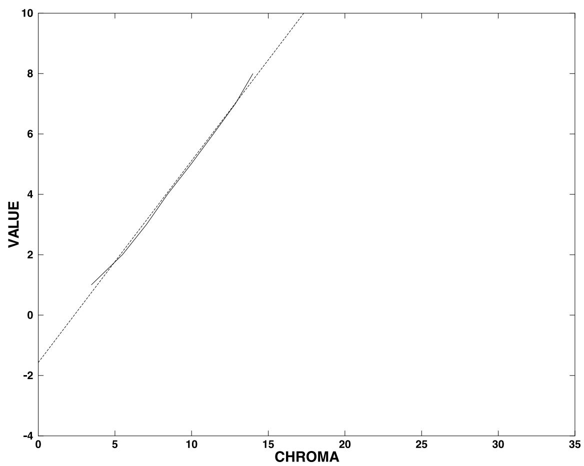

Figure 2: Chroma vs. Value Plot for Shadow Series of 6GY8/14

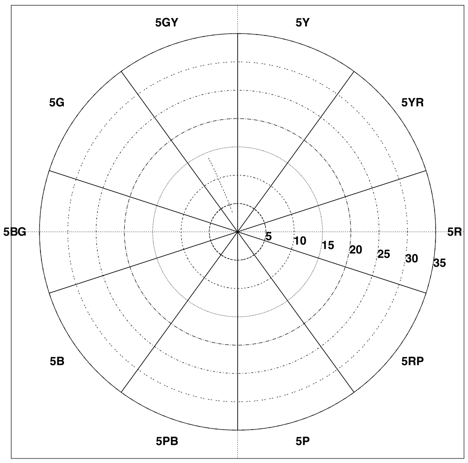

Figure 3: Chroma vs. Hue Plot for Shadow Series of 6GY8/14

shadow series, and appears in the figure, extending beyond the original shadow series.

In Figure 3, the hues and chromas of the shadow series were plotted on a polar plot. The radial direction is chroma, running from 0 to 35. The angular direction represents Munsell hue, divided into ten regions that are equally spaced perceptually. Since the shadow series decreases in value as it becomes darker, the series is lightest when it is far from the origin, and darkest when it is near the origin.

Figures 4 and 5 each plot all 182 shadow series. Figure 4 plots only chroma and value, disregarding hue. Figure 5 plots only chroma and hue, disregarding value.

3.1 Value-Chroma Relationships

The shadow series in Figure 2 seems to fall nearly on a straight line, as do the series in Figure 4. Previous researchers[6] had already noted this phenomenon, and used linear approximations. The current paper justifies the linear approximations quantitatively. Table 1 lists the values and chromas associated with the shadow series for 6GY8/14, and lists alongside them the values and chromas for the least squares line that approximates the shadow series. This least squares line is given by $V = 0.67C - 1.58$. The Munsell value differences have also been calculated.

A human observer can distinguish between 40 and 45 greys between black and white. Since the total value difference between black and white is ten value steps, human resolution is good to somewhere between 0.2 and 0.3 value steps. In other words, if two colours differ by less than 0.2 in Munsell value, and agree in hue and

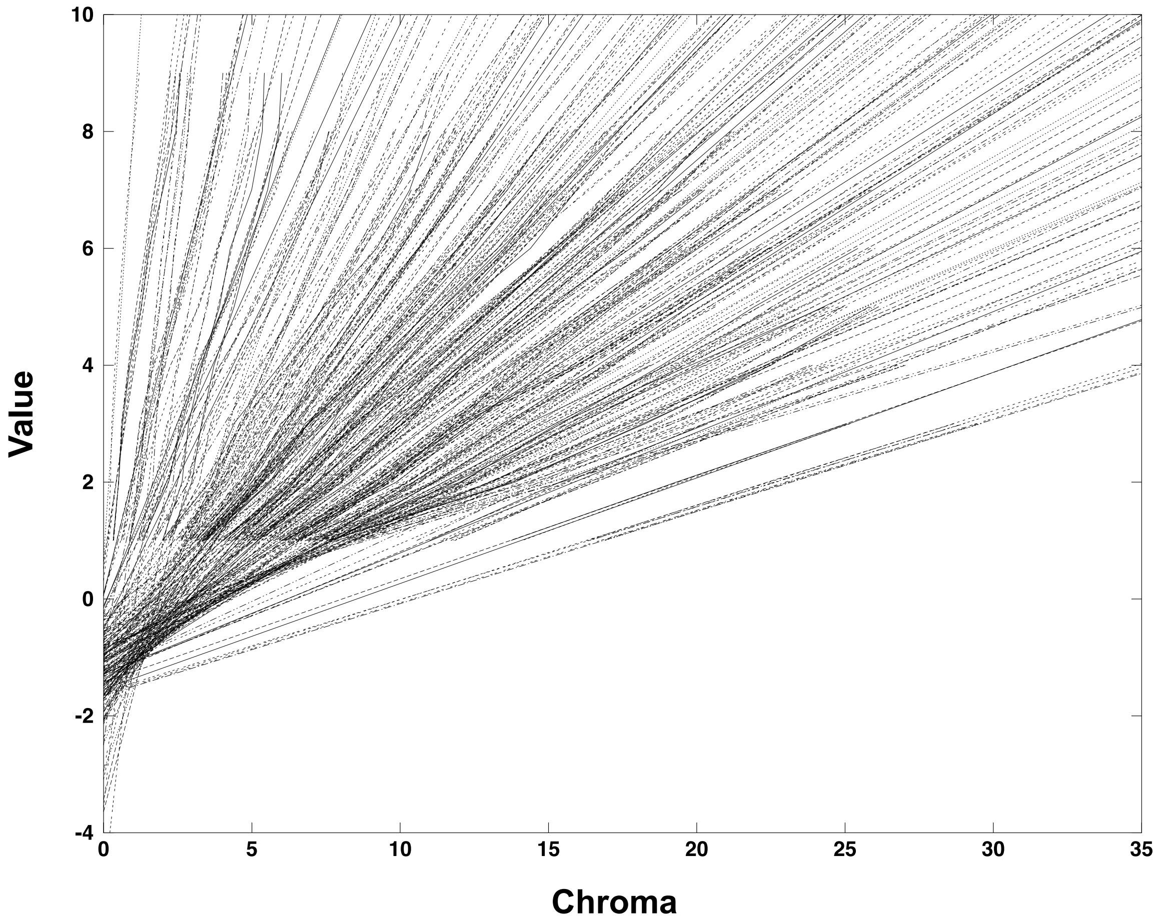

Figure 4: Chroma vs. Value Plot for Shadow Series of Optimal Colours

Inversion Chroma	Inversion Value	Least Squares Value	Value Difference
14.00	8	7.788	0.21
12.85	7	7.019	0.02
11.42	6	6.062	0.06
9.97	5	5.089	0.09
8.43	4	4.057	0.06
7.06	3	3.141	0.14
5.50	2	2.099	0.10
3.48	1	0.745	0.25

Table 1: Shadow Series and Least Squares Approximation for 6GY8/14

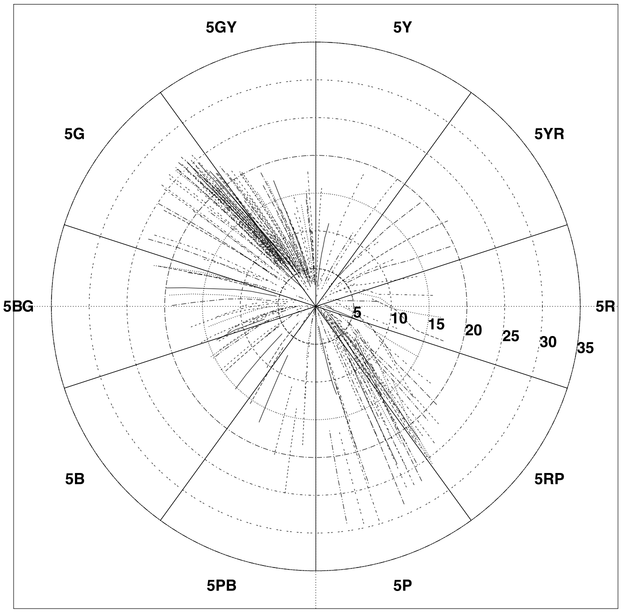

Figure 5: Chroma vs. Hue Plot for Shadow Series of Optimal Colours

chroma, then a human cannot tell them apart. This case occurs consistently in Table 1. Since the greatest value difference is 0.25, the shadow series and its least squares approximation would be indistinguishable to most observers, while a few observers might just barely notice an occasional discrepancy. Since the Munsell system reflects human visual capabilities, the least squares approximation is valid.

Similar value differences were calculated for all 1202 colours, in all 182 shadow series. Figure 6 shows a histogram, from which it can be seen that 79% of the differences are less than 0.2, 14% are between 0.2 and 0.3, and 4% are between 0.3 and 0.4. The remaining 3% are greater than 0.4, and taper off quickly. If the shadow series were all replaced with their least squares approximations, the replacements would be imperceptible 79% of the time, borderline perceptible 14% of the time, and perceptible, but minimal, in another 4%. Only about 1 out of 30 colours would show a significant difference. Given that the Munsell renotation contains a fair bit of smoothing and extrapolation, perfect agreement is not to be expected. From the very good agreement that we do see, we conclude that shadow series fall along straight lines.

Least squares lines were used because they are well documented and understood, but other approximating lines are possible. For example, one could use the line $V = 0.67C - 1.53$, instead of the line $V = 0.67C - 1.58$, for 6GY8/14. Then the

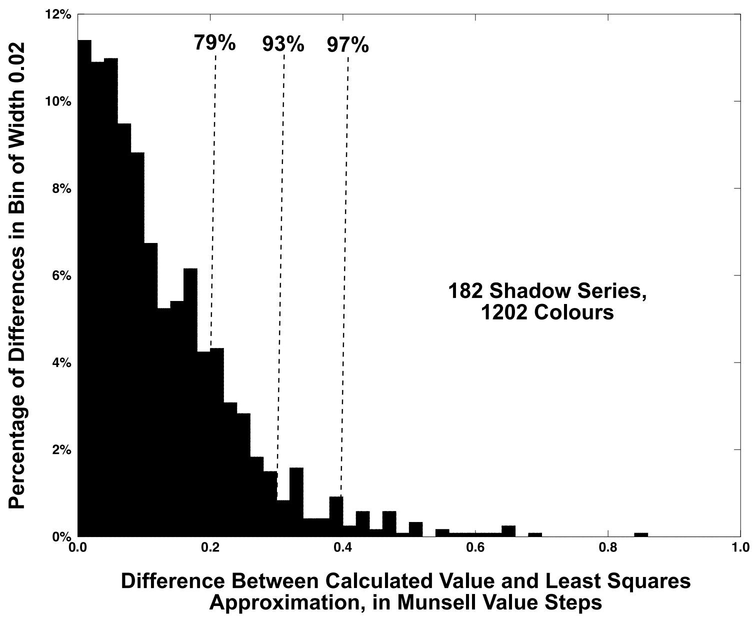

Figure 6: Value Differences in Least Squares Approximations to Shadow Series

maximum difference in Table 1 would be 0.20, instead of 0.25, so there would be no question of discrepancy. Using such lines throughout would shift the data in Figure 6 towards the left, strengthening the conclusion that shadow series fall along straight lines. In the interest of simplicity, however, the analysis in this paper used only the familiar least squares lines.

Figure 4 shows that the least squares lines tend to cross the value axis somewhere between 0 and -2. The mean y-intercept of the 182 least squares lines is -1.18, the median y-intercept is -1.16, and the standard deviation is 0.75. Extreme values of -4.97 and 0.70 are obtained, but the bulk of the crossings lie between 0 and -2.

3.2 Value-Hue Relationships

Figure 5 examines how shadows affect hues. We have already seen that chroma decreases as shadows become darker, so the shadow series in Figure 5 are all lighter far from the origin, and darker nearer the origin. If there were no change in hue as a colour became more shadowed, then each shadow series would be a straight line that would cross the origin when extended. We see that this is approximately true for most of the shadow series, but that there are also a few consistent deviations.

In particular, hues in the sector from YR to RP seem to shift counterclockwise as they become darker. A reddish colour seen in shadow would shift towards orange, which becomes brown when viewed as a darker, related colour. Similarly, orange seen in shadow would shift towards a yellowish brown. In painters' terms, these colours would be said to exhibit warm shadows. Similarly, hues in the sector from BG to PB seem to shift clockwise as they become darker, and also exhibit warm shadows, although the effect is not as pronounced as in the orange-red sector. In general, hues in shadow tend to move slightly towards the warmer, yellow part of the spectrum.

3.3 Ideal Black and White in the Munsell System

An ideal black pigment would be a pigment whose reflectance spectrum is identically 0, so that it reflects no light. A colour seen in progressively darker shadow also reflects progressively less light. In the limit, when no light is hitting it, any colour would be an ideal black, regardless of its chromaticity. Every shadow series, then, should terminate in ideal black. Since shadow series in the Munsell system are straight lines, we can extrapolate them until they cross the neutral axis at zero chroma, and the crossing point should be ideal black. While there is some variation in the crossing points, they cluster around N(-1.2), so N(-1.2) seems like a good candidate for ideal black. In the Munsell renotation, however, an ideal black would have a luminance factor of 0, so its Munsell designation, according to both Newhall[1] and the ASTM[4]. should be N0. There are thus competing designations for ideal black.

A difficulty is the lack of an ideal black pigment to use when establishing the Munsell renotation. On p. 417 of his renotation paper[1], Newhall refers to a "painter," who prepared the samples for testing. Like today, even the blackest artist's pigments are not true blacks; rather, their values are approximately 1. It could be decided, *a priori*, that the Munsell renotation should assign N0 as ideal black, and N10 as ideal white. The experimenters could determine what value differences in the available greys were perceptually equal, so N1 is the same perceptual distance from N2, as N2 is from N3, N3 from N4, and so on, up through N9. Without a sample of ideal black, however, there is no way to determine how dark a grey is perceptually one tenth of the way from black to white. The assignment of the darkest black samples to N1, then, must have been a convenient approximation.

According to this approximation, the step from N2 down to N1 should equal perceptually the step from N1 to N0. Since this equality has not been verified empirically, it is still an open question. The shadow series crossing points seem to indicate that ideal black should in fact be 2.2 value steps down from N1. In practice, the position of ideal black in the Munsell system is not important because very few colorants are darker than N1. The Munsell system is thus adequate for realistic situations, and of course its perceptually equal steps are still perceptually equal.

Some research suggests that ideal black occurs perceptually when the luminance factor is very low, but not quite zero. Evans[7] noted a "black point," for experiments with coloured lights. Below a certain luminance level, and when surrounded by a brighter stimulus, observers classified a light as black, even though there was still some luminance. The black point seems to vary with chromaticity. If such a black point also applies to non-self-luminous colour samples, then a shadow series might terminate before it reaches the neutral axis. In that case, the crossing point would not indicate ideal black. Furthermore, the termination point might differ for different chromaticities, which could explain the standard deviation of 0.75 value steps in crossing points.

Similar considerations apply, though with less force, to ideal white. A pigment's colour would be ideal white if that pigment diffusely reflected 100% of the impinging light. While very few colorants are darker than N1, many colorants are lighter than N9. For example, Ralph Mayer[11] shows the reflectance curves for two chemical formulations of titanium white (pigment PW6), the standard artist's white since the early twentieth century. The reflectance curves are practically flat, indicating a neutral colour with a 96% luminance factor, greater than the 80% luminance factor associated with Munsell value 9. Despite examples of very light colours, there is no way to judge, without a sample of ideal white, that N9 is 90 percent of the way, perceptually, from ideal black to ideal white, or that the perceptual step from N8 to N9 equals the perceptual step from N9 to ideal white (which should be N10).

Further evidence that ideal white should not be mapped to N10 comes from extending shadow series past Munsell value 9. Figure 7 shows these extensions for 24 chromaticities. The extensions all shift pronouncedly to the right, and do not form a straight line with the rest of the shadow series. One possible explanation is the linear interpolation used by the renotation inversion. Since the renotation data for value 10 consists of a single point, interpolation between values 9 and 10 is tricky, and linear interpolation might be inappropriate. Another possibility, however, is that ideal white should be placed higher than N10, say at N10.3. Then the inversion results for values greater than 9 would shift back towards the left, in line with the rest of the shadow series. This question would be best resolved empirically, by human assessments of the number of value steps between N9 and very light neutral samples.

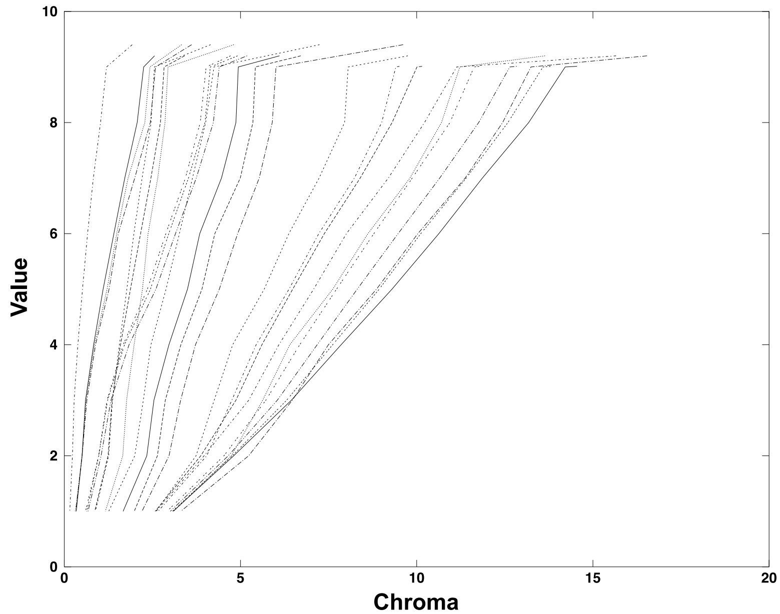

Figure 7: Shadow Series that Extend Past Munsell Value 9

4 Comparison with Previous Work

4.1 Evans's Investigations of the Late 1960s

Although shadow colours were not a primary interest, Ralph Evans's research in the late 1960's, summarized in *The Perception of Color*[7], dealt with shadows indirectly. In particular, Evans studied a circular stimulus, in the presence of a surround, usually lighter than the stimulus, and observed the results when the luminance of the stimulus varied. When the stimulus and the surround are of the same chromaticity, the stimulus could be interpreted as a shadow cast on a surface (the surround) by a circular object such as a dime. The relative luminance of the stimulus would vary with the distance of the dime, the properties of the air, the directness or diffuseness of the light source, and so on.

Lacking today's computer power, Evans's calculations were limited, and he had to rely on simple descriptive models. Today's increased processing capabilities allow a more refined look at the data. We can now directly verify some aspects of Evans's models, rather than assuming them, and replace some of his simplifications with more detailed descriptions.

Figure 9-1 of *The Perception of Color*[7] plots lines of constant chromaticity on a chroma-value plot. That figure's lines intersect the neutral axis about 1.5 value steps below 0, not far from the average crossing point of -1.2 in Figure 4. Figure 9-1 originally appeared as Figure 6 of a 1968 paper[6]. In Item (a) on p. 580 of that paper, the authors postulated both that shadow series would fall along straight lines, and, furthermore, that all those lines would intersect at a common point on the neutral axis. By visual inspection, this common point was judged to occur at N(-1.5). The current paper shows that the straight line postulate can be verified directly from the renotation data, and is thus a quantitative conclusion rather than a simplifying assumption. In addition, Evans's postulate of a common intersection point is a good approximation, but not strictly satisfied. The more detailed look in the current paper shows that the intersection point with the neutral axis can vary to within about two value steps. Furthermore, the second and third paragraphs on p. 581 of the 1968 paper[6], mention that colours of Munsell values 1 and 9 were not used, because they did not conform to the postulate of a common crossing point. The current paper includes values 1 and 9, and shows they are consistent with a varying crossing point.

In addition, p. 159 of *The Perception of Color*[7] noted that hue shifts occurred in shadow series, but did not pursue the matter any further. The current paper shows that hues tend to become yellower, as shadows become darker.

4.2 Munsell's Colorimetric Formulation

Albert Munsell originally offered two formulations of his colour system. The first was completely perceptual, involving the familiar hue, value, and chroma. The second was colorimetric, involving visual mixing on a Maxwell disk. As Tyler and Hardy[12] document, Munsell asserted that two colours, H_1V_1/C_1 and $H_1^cV_2/C_2$, would mix to grey when spun on a Maxwell disk if the disk areas they occupied were proportional to V_2C_2/V_1C_1, where H_1^c is the complement, in terms of mixtures of light, of H_1. The original Munsell system assumed that hue could be identified with dominant wavelength, so that complementary hues were defined without regard to value or chroma. The original system also assumed that $Y = V^2/100$. These two assumptions are now known to be good approximations, but not exact statements. On pp. 600-601 of a 1940 paper[13], Gibson and Nickerson show mathematically that, under these assumptions, Munsell's disk criterion implies that shadow series in the Munsell system all fall along straight lines, and all originate at N0.

Early physical Munsell exemplifications tried to satisfy both the perceptual and colorimetric formulations, but analysis, for example Fig. 8 of Gibson and Nickerson's article[13], showed that the formulations were often in conflict. In practice, colour samples were chosen by perception rather than by colorimetry. As a result, the Munsell renotation abandoned the colorimetric formulation. In his preliminary renotation report[14], Newhall documents that the human colour assessments for the renotation were solely perceptual, without any reference to Maxwell disks or colorimetry.

Since the Munsell renotation was based solely on judgements of hue, value, and chroma, with no regard for shadows, it is surprising that shadow series take such a simple form as straight lines. While it is tempting to regard their form as a legacy of Munsell's colorimetric formulation, that explanation seems unlikely, because, from the earliest days, perception took precedence over colorimetry. Even if the 1929 Munsell samples, which Newhall[1,14] took as a starting point, had contained many shadow series that were straight lines through N0, it would be remarkable for the renotation to shift series, by different value or chroma adjustments all along their length, so that their straightness was maintained, and the points at which they crossed the neutral axis were all lowered by different amounts. Instead, it seems more reasonable to disregard Munsell's colorimetric formulation, which was never adhered to strictly, and take the Munsell system as a purely perceptual system. Seen in this light, the linearity of shadow series in the Munsell systems must be taken as an observed relationship, whose explanation is currently an open question.

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